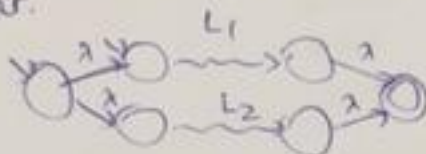


Good Morning Practice Quiz 4

Problem 1.

Given two regular languages L_1 and L_2 , show that $L_1 \cup L_2$ is always regular.

Since L_1 and L_2 are regular then both can be represented by NFAs, which can then be joined to form $L_1 \cup L_2$ as an NFA. This shows that $L_1 \cup L_2$ is also regular.



Problem 2.

Show that $L = \{a^n : n = 2^s \text{ for some positive integer } s\}$ is not regular.

Let $w = a^{2^m}$ be the word where $w \in L$ and $|w| \geq m$.

w can be decomposed as

$$w = xyz.$$

Since $|xy| \leq m$ and $|y| \geq 1$, then

$$y = a^k \quad \text{for } 1 \leq k \leq m.$$

~~then~~

$$\text{Then } w_2 = a^{2^m + k}$$

$$\text{Since } 2^m + k > 2^m \quad \text{--- (1)}$$

$$\begin{aligned} \text{and } 2^m + k &\leq 2^m + m \\ &< 2^m + 2^m \\ &= 2 \cdot 2^m = 2^{m+1} \quad \text{--- (2)} \end{aligned}$$

From (1) and (2), $w_2 \notin L$. By the pumping lemma this means that L is not regular.

Problem 1.

Show that the following grammar is ambiguous.

$$S \rightarrow AB \mid bbB$$

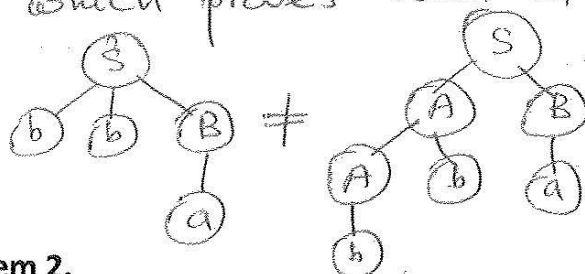
$$A \rightarrow b \mid Ab$$

$$B \rightarrow a$$

accept { Two left-most derivations
Two right-most derivations } for the same word $w \in L$
Two derivation trees \nwarrow Easier usually.

Solution:

Let $w = bba$. The following are two derivation trees which proves that G is ambiguous.



Problem 2.

Show that $L = \{a^n b^m c^k : n + m > k\}$ is context-free.

$$S \rightarrow aSc \mid aS \mid aA \mid \cancel{bB} \mid aA \mid B$$

$$A \rightarrow bAc \mid bA \mid \lambda$$

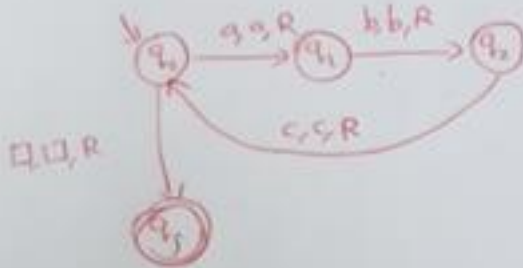
$$B \rightarrow bBc \mid bA \mid b$$

more a 's or both

more b 's

Problem 1.

Construct a Turing Machine for the language $L = \{(abc)^n : n > 0\}$.



Problem 2.

Prove that the following function is Turing computable, for all positive integers x . Use unary notation for representing integers on the tape.

